Chapter 2

1. You are given:

$$F_0(t) = 1 - \left(1 - \frac{t}{125}\right)^{\frac{1}{5}}, 0 \le t \le 125$$

Calculate:

a. $S_0(t)$
b. $\Pr[T_0 \le t]$
c. $\Pr[T_0 > t]$
d. $S_x(t)$
e. Probability that a newborn will live to age 25.
f. Probability that a person age 25 will live to age 75.
g. Probability that a person age 25 will die between age 50 and age 75.
h. ω
i. μ_x
j. μ_{25}
k. μ_{100}
l. ${}_t p_x$
m. ${}_{10} p_{50}$
n. ${}_t q_x$
o. ${}_{10} q_{50}$
p. ${}_{10} p_{50} + {}_{10} q_{50}$
q. p_{50}
r. ${}_{u|t} q_x$
s. $f_x(t)$
t. $E[T_x]$
u. $\mathring{e}_x$
v. $Var[T_x]$
w. Standard Deviation of T_{50}
x. $E[K_{120}]$
y. $Var[K_{120}]$

2. You are given that mortality follows Gompertz Law with B = 0.00027 and c = 1.1. Calculate:

a. μ_x
b. μ_{25}
c. μ_{100}
d. $S_0(t)$
e. $\Pr[T_0 \le t]$
f. $\Pr[T_0 > t]$
g. $S_x(t)$
h. Probability that a newborn will live to age 25.
i. Probability that a person age 25 will live to age 75.
j. Probability that a person age 25 will die between age 50 and age 75.
k. ω
l. ${}_t p_x$
m. ${}_{10} p_{50}$
n. ${}_t q_x$
o. ${}_{10} q_{50}$
p. ${}_{10} p_{50} + {}_{10} q_{50}$
q. p_{50}
r. ${}_{u|t} q_x$
s. $f_x(t)$

3. You are given that that $\mu_x = c$ for all $x \geq 0$ where c is a constant. This mortality law is known as a constant force of mortality.

   a. ${}_t p_x$
   b. ${}_t q_x$
   c. ω
   d. $\mathring{e}_x$
   e. $Var[T_x]$
   f. e_x
   g. ${}_{10} p_{10}$
   h. ${}_{10} p_{100}$
   i. ${}_{10} p_{500}$
   j. Would this be a reasonable model for human mortality? Why or why not?

4. You are given ${}_t q_0 = \dfrac{t^2}{10,000}$ for 0 < t < 100. Calculate:

   a. $F_0(x)$
   b. $S_0(x)$
   c. $S_x(t)$
   d. $f_0(x)$
   e. $E[T_0]$
   f. $Var[T_0]$
   g. ${}_{40} q_0$
   h. ${}_{40} p_0$
   i. $\Pr(40 < T_0 < 60)$
   j. μ_x
   k. μ_{75}
   l. ${}_t p_x$
   m. ${}_t q_x$
   n. ${}_t q_{75}$
   o. ${}_t p_{75}$
   p. $E[T_x]$
   q. $E[T_{75}]$
   r. $\mathring{e}_x$
   s. $\mathring{e}_0$
   t. $\mathring{e}_{75}$
   u. $\mathring{e}_{75:\overline{10}|}$
   v. ${}_{u|t} q_x$
   w. ${}_{10|5} q_{75}$

5. You are given that $\mu_x = \dfrac{2}{100-x}$ for $0 \le x < 100$. Calculate $F_0(x)$ and ${}_{10}p_{50}$.

6. Book Exercise 2.6

7. Book Exercise 2.13 – Submit spreadsheet to support your work.

8. You are given the following mortality table:

x	q_x for males	q_x for females
90	0.20	0.10
91	0.25	0.15
92	0.30	0.20
93	0.40	0.25
94	0.50	0.30
95	0.60	0.40
96	1.00	1.00

   a. Calculate the probability that a male exact age 91 will die at age 93 or 94.
   b. Calculate the amount that the curtate life expectancy for a female age 90 exceeds the curtate life expectancy for a male age 90.
   c. For females, calculate $e_{91:\overline{3}|}$.

9. You are given the following:
   a. $e_{40:\overline{20}|} = 18$
   b. $e_{60} = 25$
   c. ${}_{20}q_{40} = 0.2$
   d. $q_{40} = 0.003$

   Calculate e_{41}

Chapter 3

10. You are given that mortality follows the Illustrative Life Table. Calculate:
   a. ${}_{40}p_0$
   b. ${}_{40}q_0$
   c. $\Pr(60 < T_0 < 80)$
   d. ${}_{10}p_{75}$
   e. ${}_{10}q_{75}$
   f. ${}_{10|5}q_{75}$
   g. p_{80}

11. Assume that mortality follows the Illustrative Life Table for integral ages. Assume that deaths are linearly distributed (UDD) between integral ages. Calculate:

a. ${}_{0.5}q_{80}$
b. ${}_{0.5}p_{80}$
c. $\mu_{80.5}$
d. ${}_{1.5}p_{80}$
e. ${}_{1.5}q_{80}$
f. ${}_{0.5}q_{80.5}$
g. ${}_{0.5}q_{80.25}$
h. ${}_{3.2\,|\,2.4}q_{80.5}$

12. Assume that mortality follows the Illustrative Life Table for integral ages. Assume that probability of survival is linearly distributed (Constant Force) between integral ages. Calculate:

a. ${}_{0.5}q_{80}$
b. ${}_{0.5}p_{80}$
c. $\mu_{80.5}$
d. ${}_{1.5}p_{80}$
e. ${}_{1.5}q_{80}$
f. ${}_{0.5}q_{80.5}$
g. ${}_{0.5}q_{80.25}$
h. ${}_{3.2\,|\,2.4}q_{80.5}$

13. You are given $q_{80} = 0.06$ and $q_{81} = 0.09$. Calculate:

a. ${}_{0.5}q_{80}$ given UDD
b. ${}_{0.5}q_{80}$ given CFM
c. ${}_{0.5}q_{80.75}$ given UDD
d. ${}_{0.5}q_{80.75}$ given CFM

14. Complete the following table:

x	q_x	l_x	d_x
50		20,000	800
51			
52		18,000	
53	0.100		
54	0.125	14,985	

15. You are given that ${}_{t|}q_x = 0.05$ for $t = 0,1,2,...,19$.

Calculate ${}_4q_{x+8}$.

16. You are given the following select and ultimate mortality table of q_x's.

$[x]$	$q_{[x]}$	$q_{[x]+1}$	$q_{[x]+2}$	q_{x+3}	$x+3$
50	0.020	0.031	0.043	0.056	53
51	0.025	0.037	0.050	0.065	54
52	0.030	0.043	0.057	0.072	55
53	0.035	0.049	0.065	0.091	56
54	0.040	0.055	0.076	0.113	57
55	0.045	0.061	0.090	0.140	58

Calculate:

a. $p_{[54]}$

b. $p_{[53]+1}$

c. $p_{[52]+2}$

d. $p_{[51]+3}$

e. p_{54}

f. ${}_5p_{[54]}$

g. ${}_{2|2}q_{[52]}$

h. A life policy insurance policy was issued two years ago to (52). Calculate the probability that this person will live to age 59.

i. Rose is 54 and just purchased a life insurance policy. Jeff is 54 and purchased a life insurance policy at age 50. How much larger is the probability that Jeff will die during the next 4 years than the probability that Rose will die.

17. *You are given:

i. $\mu_x = F + e^{2x}, x \geq 0$

ii. ${}_{0.4}p_0 = 0.5$

Calculate F.

18. *For a certain mortality table, you are given:

i. $\mu_{80.5} = 0.0202$

ii. $\mu_{81.5} = 0.0408$

iii. $\mu_{82.5} = 0.0619$

iv. Deaths are uniformly distributed between integral ages.

Calculate ${}_{2}q_{80.5}$

19. You are given the following select mortality table.

$[x]$	$q_{[x]}$	$q_{[x]+1}$	$q_{[x]+2}$	q_{x+3}	$x+3$
90	0.04	0.10	0.17	0.20	93
91	0.06	0.14	0.18	0.30	94
92	0.08	0.16	0.27	0.40	95
93	0.14	0.24	0.36	0.50	96
94	0.21	0.32	0.45	0.70	97
95	0.28	0.40	0.63	0.90	98
96	0.35	0.56	0.81	1.00	99

Calculate $e_{[94]}$ and e_{94}.

You are given the following select and ultimate mortality table of q_x 's to be used for Numbers 20-22.

$[x]$	$q_{[x]}$	$q_{[x]+1}$	$q_{[x]+2}$	q_{x+3}	$x+3$
50	0.020	0.031	0.043	0.056	53
51	0.025	0.037	0.050	0.065	54
52	0.030	0.043	0.057	0.072	55
53	0.035	0.049	0.065	0.091	56
54	0.040	0.055	0.076	0.113	57
55	0.045	0.061	0.090	0.140	58

20. If deaths are uniformly distributed between integral ages, calculate ${}_{1.5}q_{[53]+2}$.

21. If $l_{[51]} = 100{,}000$, calculate $l_{[50]}$.

22. Teach Life Insurance Company has two cohorts of policyholders.

Cohort A has 1000 insured lives who are all age 53 and were just underwritten today.

Cohort B has 1000 insured lives who are all age 53 and were underwritten 3 years ago.

Calculate the total number of insured lives that will still be alive after 2 years.

23. For a two year select and ultimate table, you are given:

i. $q_{[x]} = 0.50q_x$

ii. $q_{[x]+1} = 0.75q_{x+1}$

Complete the following table:

$[x]$	$l_{[x]}$	$l_{[x]+1}$	l_{x+2}
105	1650	1600	1500
106	1518		1200
107			800
108			400
109			100

24. *For a 2-year select and ultimate mortality model, you are given:

i. $q_{[x]+1} = 0.80q_{x+1}$

ii. $l_{51} = 100{,}000$

iii. $l_{52} = 99{,}000$

Calculate $l_{[50]+1}$.

25. *You are given:

i. $p_x = 0.95$

ii. $p_{x+1} = 0.92$

iii. $e_{x+1.6} = 12$

iv. Deaths are uniformly distributed between ages x and $x+1$.

v. The force of mortality is constant between ages $x+1$ and $x+2$.

Calculate $e_{x+0.6}$.

Chapter 4

25.5 You are given that the interest rate is 6% and that mortality follows Makeham's law with A=0.0003, B=0.000004 and C=1.1.

Calculate the expected value and variance for a whole life insurance policy that pays a 10,000 death benefit at the moment of death for (30). Also, calculate the expected value and variance for (50).

EMAIL the spreadsheet to jeffbeckley@indy.rr.com

Chapter 4

26. You are given the following mortality table:

x	l_x	q_x	p_x	n	${}_np_{90}$
90	1000	0.10	0.90	0	1.000
91	900	0.20	0.80	1	0.900
92	720	0.40	0.60	2	0.720
93	432	0.50	0.50	3	0.432
94	216	1.00	0.00	4	0.216
95	0			5	0.000

Assume that deaths are uniformly distributed between integral ages. Calculate at $i = 4\%$:

a. A_{91}
b. $A^{1}_{91:\overline{3}|}$
c. ${}_3E_{91}$
d. $1000A_{91:\overline{3}|}$
e. ${}_{2|}A_{91}$
f. $Var[Z]$ if Z is the present value random variable for a whole life on (91).
g. $A^{(4)}_{91}$
h. $1000A^{(12)}_{93}$
i. $(IA)_{91}$
j. $(IA)^{1}_{91:\overline{3}|}$

27. Use the Illustrative Life Table with interest at 6% to calculate the following:

a. A_{50}
b. $A^{1}_{50:\overline{20}|}$
c. $A_{50:\overline{20}|}$
d. ${}_{20|}A_{50}$
e. $A^{1}_{50:\overline{35}|}$
f. $Var[Z]$ where Z is the present value random variable for A_{50}
g. $Var[Z]$ where Z is the present value random variable for $A^{1}_{50:\overline{20}|}$
h. ${}_{13}E_{40}$

28. You are given:

i. $A_x = 0.500$

ii. $A_{x+1} = 0.50617$

iii. ${}_{1|}A_x = 0.410$

Calculate q_x and i.

29. You are given:

i. $i = 8\%$

ii. $p_{90} = 0.92$

iii. Z is the present value random variable for A_{91}

iv. $Var[Z] = 0.00706$

v. ${}^2A_{91} = 0.54696$

Calculate A_{90}.

30. You are given:

i. Mortality follows the Illustrative Life Table except for age 70 where $q_{70} =$ twice the mortality rate listed in the Illustrative Life Table.

ii. $i = 6\%$

Calculate $100,000A_{70}$.

31. Assume that deaths are uniformly distributed between integral ages. Use the Illustrative Life Table with interest at 6% to calculate the following:

a. $1000\bar{A}_{50}$

b. $1000\bar{A}^{1}_{50:\overline{20|}}$

c. $1000\bar{A}_{50:\overline{20|}}$

d. $1000\,{}_{20|}\bar{A}_{50}$

e. $1000A^{(4)}_{50}$

32. Use the Illustrative Life Table with interest at 6% and the claims acceleration approach to calculate the following:

a. $1000\bar{A}_{50}$
b. $1000\bar{A}^{1}_{50:\overline{20}|}$
c. $1000\bar{A}_{50:\overline{20}|}$
d. $1000\,{}_{20|}\bar{A}_{50}$
e. $1000A^{(4)}_{50}$

33. You are given the following select and ultimate mortality table of q_x's.

$[x]$	$q_{[x]}$	$q_{[x]+1}$	$q_{[x]+2}$	q_{x+3}	$x+3$
50	0.020	0.031	0.043	0.056	53
51	0.025	0.037	0.050	0.065	54
52	0.030	0.043	0.057	0.072	55
53	0.035	0.049	0.065	0.091	56
54	0.040	0.055	0.076	0.113	57
55	0.045	0.061	0.090	0.140	58

If $i = 0.06$, calculate:

a. $A^{1}_{[54]:\overline{3}|}$
b. $A_{[54]:\overline{3}|}$

34. You are given:

i. $i = 10\%$
ii. $q_x = 0.20$
iii. $q_{x+1} = 0.40$

Calculate $A_{x:\overline{3}|}$

35. A special whole life insurance on (30) provides a death benefit of $(1.06)^t$ where t is measured from the issue date of the policy. The death benefit is payable at the moment of death.

Calculate the Expected Present Value at $i = 6\%$.

36. A special decreasing whole life insurance is issued to (60). The special whole life pays the following death benefits at the end of the year of death:

k	b_{k+1}
0-9	4000
10-19	2000
20+	1000

You are also given that mortality follows the Illustrative Life Table and $i = 0.06$.

Calculate the actuarial present value of this special whole life.

37. A special decreasing whole life insurance is issued to (45). The special whole life pays a death benefit of 5000 until age 65 and a death benefit of 2000 after age 65. The death benefit is payable at the end of the year of death.

You are given that mortality follows the Illustrative Life Table and $i = 0.06$.

Z is the present value random variable for this insurance coverage.

Calculate $E[Z]$ and $Var[Z]$.

38. A special whole life insurance policy on (80) pays a death benefit of $(1.05)^{K+1}$ where K is the number of complete years lived by (80). For example, if (80) dies between 80 and 81, then $K = 0$. If (80) dies between 81 and 82, then $K = 1$.

You are given:

i. Mortality follows the Illustrative Life Table
ii. $i = 11.3\%$

Calculate the EPV of this special whole life.

39. * For a group of individuals all age x, you are given:

    i. 25% are smokers and 75% are nonsmokers
    ii. $i = 0.02$
    iii. Mortality as follows:

k	q_{x+k} for smokers	q_{x+k} for nonsmokers
0	0.10	0.05
1	0.20	0.10
2	0.30	0.15

Calculate $10,000 A^{1}_{x:\overline{2}|}$ for an individual chosen at random from this group.

40. * A maintenance contract on a hotel promises to replace burned out light bulbs at the end of each year for three years. The hotel has 10,000 light bulbs. The light bulbs are all new. If a replacement bulb burns out, it too will be replaced with a new bulb.

You are given:

    i. For new light bulbs,

$$q_0 = 0.10$$
$$q_1 = 0.30$$
$$q_2 = 0.50$$

    ii. Each light bulb costs 1.
    iii. $i = 0.05$

Calculate the actuarial present value of this contract.

41. * A decreasing term life insurance on (80) pays $(20 - k)$ at the end of the year of death if (80) dies in year $k + 1$, for $k = 0, 1, 2, \ldots, 19$.

You are given:

    i. $i = 0.06$
    ii. For a certain mortality table with $q_{80} = 0.2$, the actuarial present value for this insurance is 13.
    iii. For the same mortality table, except that $q_{80} = 0.1$, the actuarial present value for this insurance is APV.

Calculate APV.

42. * For an increasing 10 year term insurance, you are given:

i. $b_{k+1} = 100,000(1+k), k = 0,1,2,...,9$

ii. Benefits are payable at the end of the year of death.

iii. Mortality follows the Illustrative Life Table

iv. $i = 0.06$

v. The actuarial present value of this insurance on (41) is 16,736.

Calculate the actuarial present value for this insurance on (40).

43. * For a special 3 year term insurance on (x), you are given:

i. Z is the present-value random variable for this insurance.

ii. $q_{x+k} = 0.02(k+1), k = 0,1,2$

iii. The following benefits are payable at the end of the year of death:

k	b_{k+1}
0	300
1	350
2	400

iv. $i = 0.06$

Calculate $Var[Z]$.

44. Shyu Insurance Company issues 100 whole life policies to independent lives each age 65. The death benefit for each life is 10,000 payable at the end of the year of death.

Mortality follows the Illustrative Life Table and $i = 6\%$.

Assuming the normal distribution, calculate the amount that Shyu must have on hand at time 0 to be 90% certain that the company can cover the future death benefits.

45. Li Life Insurance Company issues 400 20 year term policies to independent lives each age 70. The death benefit is 100,000 payable at the moment of death.

    Mortality follows the Illustrative Life Table and $i = 6\%$.

    At the time these policies are issued, Li sets aside 20 million.

    Assuming the normal distribution, calculate the probability that Li will have sufficient funds to pay all the death benefits.

46. Z is the present value random variable for a whole life of 1000 to (55).

    Mortality follows the Illustrative Life Table and $i = 6\%$.

    Calculate $\Pr(Z < E[Z])$.

47. Exercise 4.6 from the book – (a) and (c) only.
48. Exercise 4.10 from the book
49. Exercise 4.11 from the book

Chapter 5

50. You are given the following mortality table:

x	l_x	q_x	p_x
90	1000	0.10	0.90
91	900	0.20	0.80
92	720	0.40	0.60
93	432	0.50	0.50
94	216	1.00	0.00
95	0		

Assume that deaths are uniformly distributed between integral ages. Calculate at $i = 4\%$:

a. $\ddot{a}_{91}$

b. a_{91}

c. $\ddot{a}_{91:\overline{3}|}^{1}$

d. $Var[Y]$ if Y is the present value random variable for an annual whole life annuity due on (91).

e. $\ddot{a}_{\overline{\overline{91:3}|}}$

f. $Var[Y]$ if Y is the present value random variable for an annual 3 year temporary life annuity due on (91).

g. ${}_{2|}\ddot{a}_{91}$

h. $\bar{a}_{91}$

i. $Var[Y]$ if Y is the present value random variable for a continuous whole life annuity on (91).

j. $(I\ddot{a})_{91}$

k. $(I\ddot{a})_{91:\overline{3}|}$

51. Using the Illustrative Life Table with $i = 0.06$, calculate:

a. $\ddot{a}_{60}$

b. a_{60}

c. ${}_{10|}\ddot{a}_{60}$

d. $\ddot{a}_{60:\overline{20|}}$

e. $\bar{a}_{80}$ *assuming UDD between integer ages*

f. $\bar{a}_{50:\overline{20|}}$ *assuming UDD between integer ages*

g. $\ddot{a}_{\overline{60:\overline{10|}}}$

h. $\ddot{a}_{\overline{60:\overline{13|}}}$

i. $\ddot{a}_{60}^{(12)}$ assuming UDD between integral ages.

j. $\ddot{a}_{60}^{(12)}$ estimated using α and β formula.

k. $\ddot{a}_{60}^{(12)}$ estimated using Woolhouse formula to three terms and estimating μ_x

l. $Var[Y]$ where Y is the present value random variable for $\ddot{a}_{60}$.

m. $Var[Y]$ where Y is the present value random variable for $\ddot{a}_{60:\overline{20|}}$.

52. You are given that a continuous whole life annuity to (50) pays at a rate of 100 per year for the first 20 years and 500 per year thereafter. Calculate the actuarial present value if mortality follows the Illustrative Life Table with $i = 0.06$.

53. Peter retires from Purdue at age 60. He receives a monthly pension of 800 per month. The payments are guaranteed for 5 years.

Your are given:

a. Mortality follows the Illustrative Life Table

b. $i = 6\%$

Calculate the present value of Peter's retirement benefit.

54. You are given the following select and ultimate mortality table of q_x's.

$[x]$	$q_{[x]}$	$q_{[x]+1}$	$q_{[x]+2}$	q_{x+3}	$x+3$
50	0.020	0.031	0.043	0.056	53
51	0.025	0.037	0.050	0.065	54
52	0.030	0.043	0.057	0.072	55
53	0.035	0.049	0.065	0.091	56
54	0.040	0.055	0.076	0.113	57
55	0.045	0.061	0.090	0.140	58

If $i = 0.06$, calculate:

a. $\ddot{a}_{[54]:\overline{3}|}$

b. $a_{[54]:\overline{3}|}$

55. Mortality follows the Illustrative Life except for age 90. For age 90, $q_{90} = 0.15$. Calculate $\ddot{a}_{90}$ at $i = 0.06$.

56. Problem 5.3 in the book.

57. Problem 5.4 in the book.

58. Problem 5.5 in the book.

59. You are given:

i. $a_x = 9$

ii. $A_x = 0.6$

iii. Deaths are uniformly distributed between integral ages.

Calculate $1000\bar{A}_x$.

60. You are given:

    i. $\ddot{a}_{\overline{\overline{x:n}|}} = 22.9$

    ii. $\ddot{a}_{x:\overline{n}|} = 8$

    iii. $\ddot{a}_x = 20$

    iv. $i = 0.05$

    Calculate n.

61. Your boss has asked you to use the Illustrative Life Table with interest at 6% to estimate $100{,}000\bar{A}_{85}$.

    a. Determine an estimate assuming UDD.

    b. Determine an estimate using Woolhouse's formula with three terms. Use the values of p_x to estimate μ_x.

62. * For a special 3-year temporary life annuity-due on (x), you are given:

    i. $i = 0.06$

    ii.

t	Annuity Payment	p_{x+t}
0	15	0.95
1	20	0.90
2	25	0.85

Calculate the variance of the present value random variable for this annuity.

63. A life annuity due payable to (60) pays annual payments of 1000.

    You are given:

    i. Mortality follows the Illustrative Life Table.

    ii. $i = 6\%$

    iii. Y is the present value random variable for this annuity.

    Calculate the probability that Y is greater than the expected value of Y plus the standard deviation of Y.

64. A life annuity due payable to (40) pays monthly payments of 100.

You are given:

i. Mortality follows the Illustrative Life Table.
ii. $i = 6\%$
iii. Deaths are uniformly distributed between integral ages.
iv. Y is the present value random variable for this annuity.

Calculate the probability that Y is greater than 12,000.

Chapter 6

65. A whole life policy for 50,000 is issued to (65). The death benefit is payable at the end of the year of death. The level premiums are payable for the life of the insured.

    You are given:

    a. Mortality follows the Illustrative Life Table.
    b. $i = 6\%$.
    c. Deaths are uniformly distributed between integer ages.
    d. The equivalence principle applies.

    For this life insurance:

    a. Calculate the level annual net premium payable at the beginning of each year.
    b. Write an expression for the loss at issue random variable L_0^n
    c. Calculate the $Var[L_0^n]$.
    d. Calculate the monthly net premium payable at the beginning of each month.

66. A whole life policy for 50,000 is issued to (75). The death benefit is payable at the moment of death. The premiums are payable for the life of the insured.

    You are given:

    a. Mortality follows the Illustrative Life Table.
    b. $i = 6\%$.
    c. Deaths are uniformly distributed between integer ages.
    d. The equivalence principle applies.

    For this life insurance:

    a. Calculate the net level premium payable continuously.
    b. Write an expression for the loss at issue random variable L_0^n
    c. Calculate the $Var[L_0^n]$.

67. A 25 year endowment policy for 25,000 is issued to (40). The death benefit is payable at the end of the year of death. The level premiums are payable for the life of the insured during the term of the policy.

You are given:

a. Mortality follows the Illustrative Life Table.
b. $i = 6\%$.
c. Deaths are uniformly distributed between integer ages.
d. The equivalence principle applies.

For this endowment insurance:

a. Calculate the level annual net premium payable at the beginning of each year.
b. Write an expression for the loss at issue random variable L_0^n
c. Calculate the $Var[L_0^n]$.
d. Calculate the monthly net premium payable at the beginning of each month.

68. Tiannan buys a Term to Age 65. Tiannan is age 35. The term policy pays a death benefit of 500,000 immediately upon Tiannan's death. Level premiums are payable for 15 years.

You are given:

a. Mortality follows the Illustrative Life Table.
b. $i = 6\%$.
c. Deaths are uniformly distributed between integer ages.
d. The equivalence principle applies.

Calculate the monthly net premium payable at the beginning of each month.

69. Brittany age 25 purchases an annuity due that a monthly benefit of 1000 for as long she lives with the first payment made today.

You are given:

a. Mortality follows the Illustrative Life Table.
b. $i = 6\%$.
c. Deaths are uniformly distributed between integer ages.
d. The equivalence principle applies.

Calculate the net single premium that Brittany would pay to purchase this annuity.

70. Alex, age 20, purchases a deferred life annuity. The life annuity will pay an annual benefit of 100,000 beginning at age 65. Alex will pay a level annual net premium of P for the next 10 years to pay for this annuity.

You are given:

a. Mortality follows the Illustrative Life Table.
b. $i = 6\%$.
c. Deaths are uniformly distributed between integer ages.
d. The equivalence principle applies.

Calculate P.

71. You are given the following mortality table:

x	l_x	q_x	p_x
90	1000	0.10	0.90
91	900	0.20	0.80
92	720	0.40	0.60
93	432	0.50	0.50
94	216	1.00	0.00
95	0		

Assume that deaths are uniformly distributed between integral ages and that the equivalence principle applies. Calculate at $i = 4\%$:

a. The level annual premium for a whole life of 5000 to (90). The death benefit is payable at the end of the year of death and the premium is payable for life.
b. The variance of the loss at issue random variable for the insurance in a.
c. The monthly premium for a whole life of 5000 to (90). The death benefit is payable at the moment of death and the premium is payable for two years during the insured's lifetime.

72. Problem 6.1 in the book.

73. *Matthew and Lingxiao each purchase a fully discrete 3-year term insurance of 100,000. Matthew and Lingxiao are each 21 years old at the time of purchase.

You are given:

i. The symbol μ_{21+t} is the force of mortality consistent with the Illustrative Life Table for $t \geq 0$.

ii. Lingxiao is a standard life and her mortality follows the Illustrative Life Table.

iii. Matthew is a substandard life and has a force of mortality equal to μ'_{21+t} where $\mu'_{21+t} = \mu_{21+t} + 0.05$.

iv. $i = 5\%$ (Note that $i \neq 6\%$)

Calculate the difference between the annual benefit premium for Matthew and the annual benefit premium for Lingxiao.

74. *Amy who is 25 years old purchases a 3-yer term insurance with a death benefit of 25,000.

You are given that mortality follows the select and ultimate mortality table below

$[x]$	$l_{[x]}$	$l_{[x]+1}$	l_{x+2}	$x+2$
25	1100	1060	1000	27
26	1020	970	900	28
27	940	880	800	29

You are also given:

i. The death benefit is payable at the end of the year of death.

ii. Level premiums are payable at the beginning of each quarter.

iii. Deaths are uniformly distributed over each year of age.

iv. $i = 6\%$

Calculate the amount of each quarterly benefit premium.

75. Emily, (40), purchases a whole life policy. The policy pays a death benefit of 50,000 at the end of the year of death if Emily dies prior to age 65. It pays a death benefit of 25,000 at the end of the year of death if Emily dies after age 65.

Additionally, the policy pays a pure endowment of 25,000 if Emily survives to age 65.

Emily will pay annual benefit premiums for this policy. The annual benefit premium during the first 10 years is P. The annual benefit premium thereafter is $2P$.

You are given that mortality follows the Illustrative Life Table with $i = 6\%$.

Calculate P.

76. A whole life policy on (60) pays a death benefit of 40,000 at the moment of death. Premiums are paid annually for as long as the insured lives.

You are given:

a. Mortality follows the Illustrative Life Table.
b. $i = 0.06$
c. Commissions are 80% of premiums in the first year and 5% of premiums thereafter.
d. The issue expenses at time zero are 300 per policy.
e. The renewal expense at the beginning of each year beginning with the second year is 25.

Calculate the gross premium for this policy using the equivalence principle.

77. A whole life policy on (80) pays a death benefit of 10,000 at the end of the year of death. Premiums are paid annually for as long as the insured lives.

You are given:

a. Mortality follows the Illustrative Life Table.
b. $i = 0.06$
c. Commissions are c% of premiums in the first year and 5% of premiums thereafter.
d. The issue expenses at time zero are 300 per policy.
e. The renewal expense at the beginning of each year beginning with the second year is 25.
f. The gross premium for this policy using the equivalence principle is 1279.21.

Calculate c.

78. Cong Actuarial Consulting provides a life insurance benefit to Candace who is an consultant age 40. If Candace dies after age 60, a death benefit of 100,000 will be paid at the end of the year of death.

Cong will pay level annual benefit premiums for during the deferral period of 20 years. No premiums are payable after 20 years.

You are given:

i. Mortality follows the Illustrative Life Table.
ii. $i = 0.06$.
iii. The gross premium is 125% of the annual benefit premium.
iv. Commissions are 25% in the first year and 5% thereafter. No commissions are paid after the premiums stop.
v. There is a per policy expense of 110 in the first year and 50 each year thereafter.
vi. L_0 is the present value of future losses at issue random variable.

Calculate $E[L_0]$.

79. A 20 year term insurance policy is issued to (70) with a death benefit of 1,000,000 payable at the end of the year of death. Premiums are paid annually during the term of the policy.

You are given:

a. Mortality follows the Illustrative Life Table.
b. $i = 0.06$
c. Commissions are 50% of premiums in the first year and 7% of premiums thereafter.
d. The issue expenses at time zero are 1000 per policy plus 1 per 1000 of death benefit.
e. The renewal expense at the beginning of each year including the first year is 40.
f. A termination expense of 500 is incurred at the end of the year of death.

Calculate the gross premium for this policy using the equivalence principle.

80. A special 30 year term policy on (35) provides a death benefit that is paid at the end of the year of death. The death benefit is 300,000 for death during the first 10 years of the policy. The death benefit is 200,000 if the insured dies after 10 years, but before 20 years. The death benefit is 100,000 if the insured dies during the last 10 years of the policy.

    Gross premiums are payable annually for the term of the policy. The annual gross premium is $3G$ during the first 10 years, $2G$ during the second 10 years, and G during the last 10 years.

    You are given:

    i. Mortality follows the Illustrative Life Table.
    ii. $i = 0.06$
    iii. Commissions are 50% of the premium in the first year and 5% thereafter.
    iv. Maintenance expenses are 50 per year payable at the start of every year.
    v. The issue expense is 400 payable at issue.
    vi. The gross premium is determined using the equivalence principle.

    Determine G.

81. A whole life policy with a death benefit of 250,000 is issued to (35). The death benefit is payable at the end of the year of death. Annual premiums are payable for the life of the insured and are determined using the equivalence principle.

    You are given:

    a. Mortality follows the Illustrative Life Table.
    b. $i = 6\%$
    c. Issue expenses incurred at the time of issue are 300.
    d. Maintenance expenses incurred at the start of each policy year (including the first year) are 50.

    Calculate

    a. The gross premium using the equivalence principle.
    b. The probability that this policy will result in a profit.
    c. The gross premium using the Portfolio Percentile Premium Principle assuming the company issues 10,000 policies and wants the probability of a profit to be 95%.

82. You are given the following mortality table:

x	l_x	q_x	p_x
90	1000	0.10	0.90
91	900	0.20	0.80
92	720	0.40	0.60
93	432	0.50	0.50
94	216	1.00	0.00
95	0		

For a whole life to (91) with a death benefit of 10,000 payable at the end of the year of death and level annual premiums, the expenses are 200 per policy at issue and 40 per policy at the beginning of each year including the first year.

You are given that $i = 4\%$.

a. Calculate the level gross premium using the equivalence principle.
b. Complete the following table:

K_x	L_0^g	Probability
0		
1		
2		
3		

c. The variance of the loss at issue random variable.
d. Calculate the expected value and the variance of the loss at issue random variable if the gross premium was 4000.

83. A whole life policy on (60) pays a death benefit of 400,000 at the end of the year of death. Premiums are paid annually for as long as the insured lives.

You are given:

a. Mortality follows the Illustrative Life Table.
b. $i = 0.06$
c. Commissions are 80% of premiums in the first year and 5% of premiums thereafter.
d. The issue expenses at time zero are 300 per policy.
e. The renewal expense at the beginning of each year beginning with the second year is 25.

Calculate the gross premium for this policy using the portfolio percentile premium principle assuming 100 policies and a probability of a profit of 80%.

Chapter 7

84. You are given that Mortality follows the Illustrative Life Table with $i = 6\%$. Assume that mortality is uniformly distributed between integral ages. Calculate:

    a. Calculate ${}_{10}V^n$ for a whole life policy issued to (60). The death benefit is 100,000 and is payable at the end of the year of death. The insurance has level annual benefit premiums payable for the life of the insured.
    b. Calculate ${}_{10}V^n$ for a 20 year term insurance issued to (40). The death benefit is 250,000 and is payable at the end of the year of death. The insurance has level annual benefit premiums payable for 20 years during the life of the insured.
    c. Calculate ${}_{5}V^n$ and ${}_{10}V^n$ for an Endowment to 65 issued to (35). The death benefit is 10,000 and is payable at the end of the year of death. The insurance has level annual premiums payable for 10 years during the life of the insured.
    d. Calculate ${}_{10}V^n$ for an annuity issued to (65). The annuity has level annual payments during the life of the annuitant of 1000. The annuity was purchased for a single premium. Calculate ${}_{10}V^n$ both immediately before the annuity payment at age 75 and immediately after the annuity payment at 75.
    e. Calculate ${}_{10}V^n$ for a whole life policy issued to (60). The death benefit is 50,000 and is payable at the moment of death. The insurance has level annual premiums payable for the life of the insured.
    f. Calculate ${}_{10}V^n$ for a whole life policy issued to (60). The death benefit is 50,000 and is payable at the moment of death. The insurance has level monthly premiums payable for the life of the insured.
    g. Calculate ${}_{10}V^{FPT}$ (the modified premium reserve using the Full Preliminary Term method) for a whole life policy issued to (60). The death benefit is 100,000 and is payable at the end of the year of death. The insurance has level annual benefit premiums payable for the life of the insured.
    h. Calculate ${}_{10}V^{FPT}$ (the modified premium reserve using the Full Preliminary Term method) for a 20 year term insurance issued to (40). The death benefit is 250,000 and is payable at the end of the year of death. The insurance has level annual benefit premiums payable for 20 years during the life of the insured.
    i. Calculate ${}_{5}V^{FPT}$ and ${}_{10}V^{FPT}$ (the modified premium reserves using the Full Preliminary Term method) for an Endowment to 65 issued to (35). The death benefit is 10,000 and is payable at the end of the year of death. The insurance has level annual premiums payable for 10 years during the life of the insured.

85. You are given that Mortality follows the Illustrative Life Table with $i = 6\%$. Assume that mortality is uniformly distributed between integral ages.

The expenses associated with the following policies is:

a. Commissions of 50% of premium in the first year and 8% of premium thereafter
b. 100 per policy at the beginning of the first year
c. 25 per policy at the beginning of every year including the first
d. 250 at the time that a claim payment is made or upon the payment of an endowment

Calculate:

a. Calculate ${}_{10}V^g$ for a whole life policy issued to (60). The death benefit is 100,000 and is payable at the end of the year of death. The insurance has level annual gross premiums payable for the life of the insured. The gross premium is calculated using the equivalence principle.
b. Calculate ${}_0V^g$ and ${}_{10}V^g$ for a 20 year term insurance issued to (40). The death benefit is 250,000 and is payable at the end of the year of death. The insurance has level annual benefit premiums of 1600 payable for 20 years during the life of the insured.
c. Calculate ${}_5V^g$ and ${}_{10}V^g$ for an Endowment to 55 issued to (35). The death benefit is 10,000 and is payable at the end of the year of death. The insurance has level annual gross premiums payable for 10 years during the life of the insured. The gross premium was determined as $P + 50$ where P is determined using the equivalence principle. (Note: The 25 per policy expense continues for all 20 years.)

86. You are given:

i. $1000\bar{A}_x = 400$
ii. $1000\bar{A}_{x+t} = 500$

Calculate ${}_tV^n$ for a whole life insurance issued to (x) with a death benefit of 1000 payable at the moment of death. Premiums are payable continuously during (x)'s lifetime.

87. You are given:

i. $\bar{a}_x = 12$
ii. $\bar{a}_{x+t} = 8.4$

Calculate ${}_tV^n$ for a whole life insurance issued to (x) with a death benefit of 1000 payable at the moment of death. Premiums are payable continuously during (x)'s lifetime.

88. You are given the following mortality table:

x	l_x	q_x	p_x
90	1000	0.10	0.90
91	900	0.20	0.80
92	720	0.40	0.60
93	432	0.50	0.50
94	216	1.00	0.00
95	0		

For a whole life to (91) with a death benefit of 10,000 payable at the end of the year, the net premium is 3736.756. The interest rate is 4%.

Calculate ${}_tV$ for $t = 1, 2, 3, and\ 4$. The easiest way to find these reserves is using the recursive formula.

89. You are given the following mortality table:

x	l_x	q_x	p_x
90	1000	0.10	0.90
91	900	0.20	0.80
92	720	0.40	0.60
93	432	0.50	0.50
94	216	1.00	0.00
95	0		

For a special 4 year term issued to (91) which pays death benefits at the end of the year, you are given:

i. $i = 4\%$
ii. The death benefit during the first two years is 1000
iii. The death benefit during the second two years is 500
iv. The net annual premium for the first two years is twice the net annual premium for the last two years.

Calculate ${}_2V$, the net premium reserve at the end of the second year for this special term insurance.

90. You are given the following mortality table:

x	l_x	q_x	p_x	$_{x-90}p_{90}$
90	1000	0.10	0.90	1.000
91	900	0.20	0.80	0.900
92	720	0.40	0.60	0.720
93	432	0.50	0.50	0.432
94	216	1.00	0.00	0.216
95	0			

For a special 3 year endowment insurance issued to (90), you are given:

i. $i = 4\%$

ii. The death benefit payable at the end of the year of death and the endowment amount are 2000

iii. The net annual premium for the first year is twice the net annual premium for the second year which is twice the net annual premium for the third year.

If the premium at the start of the third year is 282.235, calculate the net premium reserves using the recursive formula.

91. For a whole life policy (50) with annual premiums and death benefit of 1000 payable at the end of the year of death, you are given:

a. $_9V^n = 500$

b. $_{10}V^n = 560$

c. $i = 10\%$

d. The net premium is 60.

Calculate q_{59}

92. For a whole life policy on (x) with a death benefit of 10,000 payable at the end of the year of death, you are given:
    a. The annual gross premium is calculated using the equivalence principle.
    b. The gross premium reserve at the end of the first year is -79.56. (Note: This is a negative reserve.)
    c. $q_x = 0.0020$
    d. $q_{x+1} = 0.0021$
    e. $i = 8\%$
    f. Our commissions are 50% of premium in the first year and 10% thereafter.
    g. Our per policy expenses are 100 in the first year and 20 thereafter.

    Calculate the annual gross premium.

93. A special whole life policy to (40) provides a death benefit that is 100,000 for the first ten years of the policy. Each year thereafter, the death benefit decreases. The death benefit is payable at the end of the year of death.

    The policy has a level annual premium of 700.

    You are given that mortality follows the Illustrative Life Table with $i = 6\%$.

    Calculate ${}_5V^n$.

94. A whole life insurance of 1000 is issued on (65). Death benefits are paid at the end of the year of death. Mortality follows the Illustrative Life Table with interest at 6%. Net premiums are paid annually.

    Determine ${}_{10.7}V^n$.

95. A fully discrete whole life on (70) provides a death benefit of 25,000. The annual gross premium is determined using the equivalence principle using the following assumptions:

a. Mortality follows the Illustrative Life Table.
b. $i = 0.06$
c. Commissions as a percent of premium which are 60% in the first year and 4% in renewal years.
d. The issue expense at the start of the first year is 100.
e. The annual maintenance expense is 20 at the start of each year including the first year.

The actual experience in the 6^{th} year is:

a. Mortality is 95% of the Illustrative Life Table
b. $i = 0.058$
c. The annual maintenance expense is 30.
d. Commissions and issue expenses are equal to expected.

Calculate:

a. The annual gross premium
b. The gross premium reserve at the end of 5 years
c. The gross premium reserve at the end of 6 years
d. The total gain in the 6^{th} year
e. Allocate the gain to mortality, interest rate, and expenses in that order.

96. *Your company issues fully discrete whole policies to a group of lives age 40. For each policy, you are given:

a. The death benefit is 50,000.
b. Assumed mortality and interest are the Illustrative Life Table at 6%.
c. Assumed gross premium is 125% of the benefit premium.
d. Assumed expenses are 5% of gross premium, payable at the beginning of each year, and 300 to process each death claim, payable at the end of the year of death.
e. Profits are based on gross premium reserves.

During year 11, actual experience is as follows:

a. There are 1000 lives in force at the beginning of the year.
b. There are five deaths.
c. Interest earned equals 6%.
d. Expenses equal 6% of gross premiums and 100 to process each death claim.

For year 11, you calculate the gain due to mortality and then the gain due to expenses.

Calculate the gain due to expenses during year 11.

97. Norris Life sells a three year term insurance policy with a death benefit of 10,000 to (x). Annual premiums are payable for three years. Death benefits are assumed to be paid at the end of the year. You are given the following:

i. All expenses occur at the beginning of the year.
ii. Interest is 8%.
iii.

Year	Mortality	Per Policy Expense	Percent of Premium Expense
1	0.010	130	20%
2	0.015	30	8%
3	0.020	30	8%

iv. The gross premium is 300.

Calculate the Asset Share at t = 0, 1, 2, and 3

98. * For a whole life insurance on (40), you are given:

i. The premium is payable continuously at a level annual premium rate of 66, payable for the first 20 years.

ii. The death benefit payable at the moment of death is 2000 for the first 20 years and 1000 thereafter.

iii. $\delta = 0.06$

iv. $1000\bar{A}_{50} = 333.33$

v. $1000\bar{A}^{1}_{50:\overline{10}|} = 197.81$

vi. $1000\,{}_{10}E_{50} = 406.57$

Calculate ${}_{10}V$, the net premium reserve for this insurance at time 10.

99. * For a 3-year endowment insurance of 1000 on (x):

i. The annual premium is paid at the beginning of the year and the death benefit is paid at the end of the year of death.

ii. $q_x = q_{x+1} = 0.20$

iii. $i = 0.06$

iv. The net benefit premium is 373.63

Calculate ${}_{2}V - {}_{1}V$.

100. * For a 5-payment 10-year decreasing term insurance on (60), you are given:

i. The death benefit payable at the end of year $k+1 = 1000(10-k), k = 0,1,2...,9$

ii. Level annual net premiums are payable for five years and equal 218.15 each.

iii. $q_{60+k} = 0.02 + 0.001k, k = 0,1,2,...,9$

iv. $i = 0.06$

Calculate ${}_{2}V$, the net premium reserve at the end of year 2.

101. Norris Life Insurance Company sells a whole life policy with a benefit of 1 million to (65). The death benefit is payable at the end of the year of death. The policy has level annual premiums. You are given the following assumptions:

i. Mortality follows the mortality in the Illustrative Life Table.
ii. Interest is 6% per annum.
iii. Expenses at the beginning of each year are as follows:
    1. Per policy expense is $100 in the first year and $40 per policy for each year thereafter;
    2. Percent of premium expenses are 50% in the first year and 3% thereafter;
    3. Per 1000 expense of $1.00 in the first year and $0.10 thereafter; and
    4. $200 per policy per claim.

a. Calculate the annual net premium.
b. Calculate the annual gross premium using the equivalence principle.
c. Calculate the level annual expense premium.
d. Calculate the net premium reserve at the end of the 10^{th} year.
e. Calculate the expense reserve at the end of the 10^{th} year.
f. Calculate the gross premium reserve at the end of the 10^{th} year.
g. Calculate the Asset Share at the end of the first year.
h. Calculate the Asset Share at the end of the second year.

102. Norris Life also sells a three year term insurance policy with a benefit of 10,000 to (x). Annual premiums are payable for three years. Death benefits are assumed to be paid at the end of the year. You are given the following:

i. All expenses occur at the beginning of the year.
ii. Interest is 8%.
iii.

Year	Mortality	Per Policy Expense	Percent of Premium Expense
1	0.010	130	20%
2	0.015	30	8%
3	0.020	30	8%

Calculate the gross premium using the equivalence principle.

Complete the following table:

t	${}_tV^n$	${}_tV^e$	${}_tAS$
0			
1			
2			
3			

103. For a fully continuous whole life of 100,000 on (50), you are given:

   a. The gross premium reserve at $t = 10$ is 15,000.
   b. The gross premium is paid at a rate of 2200 per year.
   c. The force of interest is 8% .
   d. $\mu_{50} = 0.01$
   e. The following expenses payable continuously:
      i. 50% of premium in the first year and 5% of premium in years 2 and later;
      ii. 40 per policy in the first year and 20 per policy in years 2 and later; and
      iii. 500 payable at the moment of death.

   Calculate the derivative of the gross premium reserve with respect to time at $t = 10$.

   Calculate the annual rate of increase of the gross premium reserve at time $t = 10$.

104. For a fully continuous whole life of 100,000 on (50), you are given:

   a. The gross premium reserve at $t = 10$ is 15,000.
   b. The gross premium is paid at a rate of 2200 per year.
   c. The force of interest is 4%.
   d. The force of mortality follows Gompertz law with $B = 0.0015$ and $c = 1.03$
   e. The following expenses payable continuously:
      iv. 50% of premium in the first year and 5% of premium in years 2 and later;
      v. 40 per policy in the first year and 20 per policy in years 2 and later; and
      vi. 500 payable at the moment of death.

   Estimate the gross premium reserve at $t = 11$ using Euler's method with $h = 0.5$.

105. For a fully continuous 10 year term insurance issued to age 70, you are given:

   a. The death benefit if 250,000.
   b. The net benefit premium is paid at a rate of 13,000 per year.
   c. The force of interest is 6%.
   d. The force of mortality follows Makeham's law with $A = 0.005$, $B = 0.002$ and $c = 1.05$

   Estimate the net premium reserve at $t = 9.5$ using Euler's method with $h = 0.25$.

106. For a fully continuous 10 endowment insurance issued to age 70, you are given:

   a. The death benefit if 250,000.
   b. The net benefit premium is paid at a rate of P per year.
   c. The force of interest is 6%.
   d. The force of mortality follows Makeham's law with $A = 0.005$, $B = 0.002$ and $c = 1.05$
   e. The net premium reserve at $t = 9.5$ using Euler's method with $h = 0.25$ is estimated to be 230,000.

   Calculate P.

107. A whole life policy of 100,000 on (60) has a death benefit payable at the end of the year. The policy has level annual premiums for the life of the insured.

   You are given that mortality follows the Illustrative Life Table with interest at 6%.

   Calculate:

   a. The first year net premium under Full Preliminary Term.
   b. The net premium under Full Preliminary Term for renewal years (years 2 and later).
   c. Calculate the ${}_{10}V^{FPT}$, the modified net premium reserve at the end of 10 years.
   d. Calculate the ${}_{0.7}V^{FPT}$, the modified net premium reserve at time 0.7.
   e. Calculate the ${}_{10.7}V^{FPT}$, the modified net premium reserve at the end of 10 years.

108. A 20 Year Term policy of 500,000 on (40) has a death benefit payable at the end of the year. The policy has level annual premiums for the life of the insured.

   You are given that mortality follows the Illustrative Life Table with interest at 6%.

   Calculate:

   a. The first year net premium under Full Preliminary Term.
   b. The net premium under Full Preliminary Term for renewal years (years 2 and later).
   c. Calculate the ${}_{10}V^{FPT}$, the modified net premium reserve at the end of 10 years.

109. A 20 Endowment policy of 20,000 on (65) has a death benefit payable at the end of the year. The policy has level annual premiums for the life of the insured.

You are given that mortality follows the Illustrative Life Table with interest at 6%.

Calculate:

a. The first year net premium under Full Preliminary Term.
b. The net premium under Full Preliminary Term for renewal years (years 2 and later).
c. Calculate the ${}_{10}V^{FPT}$, the modified net premium reserve at the end of 10 years.

110. *For a fully discrete whole life insurance of 1000 on (80):

a. $i = 0.06$
b. $\ddot{a}_{80} = 5.89$
c. $\ddot{a}_{90} = 3.65$
d. $q_{80} = 0.077$

Calculate ${}_{10}V^{FPT}$, the full preliminary term reserve for this policy at the end of year 10.

111. *A special fully discrete 3-year endowment insurance on (x) pays a death benefit of 25,000 if death occurs during the first year, 50,000 if death occurs during the 2nd year and 75,000 if death occurs during the 3rd year.

You are given:

a. The maturity benefit is 75,000.
b. Annual benefit premiums increase 20% per year, compounded annually.
c. $i = 8\%$
d. $q_x = 0.08$
e. $q_{x+1} = 0.10$

Calculate ${}_2V$, the benefit reserve at the end of year 2.

Chapter 8

112. For a multiple state model where there are two states:

    i. State 0 is a person is alive
    ii. State 1 is a person is dead

Further you are given that a person can transition from State 0 to State 1 but not back again.

Using the Illustrative Life Table, determine the following:

a. ${}_{10}p_{80}^{00}$
b. ${}_{10}p_{80}^{01}$
c. ${}_{10}p_{80}^{10}$

113. For a multiple state model, there are three states:

    i. State 0 is a person is healthy
    ii. State 1 is a person is permanently disabled
    iii. State 2 is a person is dead

Further you are given that a person can transition from State 0 to State 1 or State 2. Further, a person in State 1 can transition to State 2, but not to State 0. Finally, a person in State 2 cannot transition.

You are given the following transitional intensities:

i. $\mu_x^{01} = 0.05$
ii. $\mu_x^{02} = 0.01$
iii. $\mu_x^{12} = 0.02$

Calculate the following:

a. ${}_{10}p_x^{00}$
b. ${}_{10}p_x^{01}$
c. ${}_{10}p_x^{02}$

114. For a multiple state model, there are three states:

i. State 0 is a person is healthy
ii. State 1 is a person is sick
iii. State 2 is a person is dead

Further you are given that a person can transition from State 0 to State 1 or State 2. Further, a person in State 1 can transition to State 0 or to State 2. Finally, a person in State 2 cannot transition.

You are given the following transitional intensities:

i. $\mu_x^{01} = 0.05$
ii. $\mu_x^{10} = 0.03$
iii. $\mu_x^{02} = 0.01$
iv. $\mu_x^{12} = 0.02$

Calculate the following:

a. ${}_{10}p_x^{\overline{00}}$
b. Assuming that only one transition can occur in any monthly period, use the Euler method to calculate:
   i. ${}_0p_x^{00}$
   ii. ${}_0p_x^{01}$
   iii. ${}_{1/12}p_x^{00}$
   iv. ${}_{1/12}p_x^{01}$
   v. ${}_{2/12}p_x^{00}$
   vi. ${}_{2/12}p_x^{01}$

115. For a multiple state model, there are three states:

i. State 0 is a person is healthy
ii. State 1 is a person is sick
iii. State 2 is a person is dead

Further you are given that a person can transition from State 0 to State 1 or State 2. Also, a person in State 1 can transition to State 0 or to State 2. Finally, a person in State 2 cannot transition.

You are given the following transitional intensities:

i. $\mu_{x+t}^{01} = 0.05 + .001t$
ii. $\mu_x^{10} = 0.03 - .0005t$
iii. $\mu_x^{02} = 0.01$
iv. $\mu_x^{12} = 0.02$

Assume that only one transition can occur in any monthly period.

If ${}_{10}p_x^{00} = 0.90$ and ${}_{10}p_x^{01} = 0.07$, use the Euler method to calculate ${}_{121/12}p_x^{01}$.

116. *For a multiple state model, there are three states:

i. State 0 is a person is healthy
ii. State 1 is a person is sick
iii. State 2 is a person is dead

Further you are given that a person can transition from State 0 to State 1 or State 2. Also, a person in State 1 can transition to State 0 or to State 2. Finally, a person in State 2 cannot transition.

You are given the following transitional intensities:

v. $\mu_{x+t}^{01} = 0.06$
vi. $\mu_x^{10} = 0.03$
vii. $\mu_x^{02} = 0.01$
viii. $\mu_x^{12} = 0.04$

Calculate the probability that a disabled life on July 1, 2012 will become healthy at some time before July 1, 2017 but will not remain continuously healthy until July 1, 2017.

117. * Employees in Purdue Life Insurance Company (PLIC) can be in:

i. State 0: Non-Executive employee
ii. State 1: Executive employee
iii. State 2: Terminated from employment

Emily joins PLIC as a non-executive employee at age 25.

You are given:

i. $\mu^{01} = 0.008$
ii. $\mu^{02} = 0.02$
iii. $\mu^{12} = 0.01$
iv. Executive employees never return to non-employee executive state.
v. Employees terminated from employment are never rehired.
vi. The probability that Emily lives for 30 years is 0.92, regardless of state.

Calculate the probability that Emily will be an executive employee of PLIC at age 55.

118. A person working for Organized Crime Incorporated (OCI) can have one of three statuses during their employment. The three statuses are:

a. In Good Standing
b. Out of Favor
c. Dead

The following transitional probabilities indicates the probabilities of moving between the three status in an given year:

	In Good Standing	Out of Favor	Dead
In Good Standing	0.6	0.3	0.1
Out of Favor	0.2	0.3	0.5
Dead	0	0	1.0

Calculate the probability that a person In Good Standing now will be Out of Favor at the end of the fourth year.

119. A person working for Organized Crime Incorporated (OCI) can have one of three statuses during their employment. The three statuses are:

a. In Good Standing
b. Out of Favor
c. Dead

The following transitional probabilities indicates the probabilities of moving between the three status in an given year:

	In Good Standing	Out of Favor	Dead
In Good Standing	0.6	0.3	0.1
Out of Favor	0.2	0.3	0.5
Dead	0	0	1.0

At the beginning of the year, there are 1000 employees In Good Standing. All future states are assumed to be independent.

i. Calculate the expected number of deaths over the next four years.
ii. Calculate the variance of the number of the original 1000 employees who die within four years.

120. Animals species have three possible states: Healthy (row 1 and column 1 in the matrices), Endangered (row 2 and column 2 in the matrices), and Extinct (row 3 and column 3 in the matrices). Transitions between states vary by year where the subscript indicates the beginning of the year.

$$Q_0 = \begin{pmatrix} 0.80 & 0.20 & 0 \\ 0 & 0.75 & 0.25 \\ 0 & 0 & 1 \end{pmatrix}$$

$$Q_1 = \begin{pmatrix} 0.90 & 0.10 & 0 \\ 0.20 & 0.70 & 0.10 \\ 0 & 0 & 1 \end{pmatrix}$$

$$Q_2 = \begin{pmatrix} 0.95 & 0.05 & 0 \\ 0.25 & 0.70 & 0.05 \\ 0 & 0 & 1 \end{pmatrix}$$

$$Q_i = \begin{pmatrix} 0.95 & 0.05 & 0 \\ 0.3 & 0.70 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

for $i > 2$

Calculate the probability that a species endangered at time 0 will become extinct.

121. A fully continuous whole life policy to (60) is subject to two decrements – Decrement 1 is death and Decrement 2 is lapse. The benefit upon death is 1000. No benefit is payable upon lapse.

You are given:

a. $\mu_x^{(1)} = 0.015$

b. $\mu_x^{(2)} = 0.100$

c. $\delta = 0.045$

Calculate the P, the premium rate payable annually.

122. A fully continuous whole life policy to (60) is subject to two decrements – Decrement 1 is death by accident and Decrement 2 is death by any other cause. The benefit upon death by accident is 2000. The death benefit upon death by any other cause is 1000.

You are given:

a. $\mu_x^{(1)} = 0.015$

b. $\mu_x^{(2)} = 0.025$

c. $\delta = 0.06$

Calculate the P, the premium rate payable annually.

123. For a multiple state model, there are three states:

i. State 0 is a person is healthy
ii. State 1 is a person is sick
iii. State 2 is a person is dead

Further you are given that a person can transition from State 0 to State 1 or State 2. Also, a person in State 1 can transition to State 0 or to State 2. Finally, a person in State 2 cannot transition.

You are given the following matrix of transitional probabilities:

$$\begin{bmatrix} 0.80 & 0.15 & 0.05 \\ 0.60 & 0.25 & 0.15 \\ 0 & 0 & 1 \end{bmatrix}$$

A special 3-year term policy pays 500,000 at the end of the year of death. It also pays 100,000 at the end of the year if the insured is disabled.

Premiums are payable annually if the insured is healthy (state 0).

You are given $i = 10\%$.

i. Calculate the present value of the death benefits to be paid.
ii. Calculate the present value of the disability benefits to be paid.
iii. Calculate the annual benefit premium.
iv. Calculate the total reserve that would be held at the end of the first year.
v. Calculate the reserve associated with each person in state 0 at the end of the first year.
vi. Calculate the reserve associated with each person in state 1 at the end of the first year.

124. A person working for Organized Crime Incorporated (OCI) can have one of three statuses during their employment. The three statuses are:

a. In Good Standing
b. Out of Favor
c. Dead

The following transitional probabilities indicates the probabilities of moving between the three status in an given year:

	In Good Standing	**Out of Favor**	**Dead**
In Good Standing	0.6	0.3	0.1
Out of Favor	0.2	0.3	0.5
Dead	0	0	1.0

The Italian Life Insurance Company issues a special 4 year term insurance policy covering employees of OCI. The policy pays a death benefit of 10,000 at the end of the year of death.

Assume that the interest rate is 25% (remember who we are dealing with).

i. Calculate the actuarial present value of the death benefit for an employee who is In Good Standing at the issue of the policy.
ii. Calculate the annual benefit premium (paid at the beginning of the year by those in Good Standing and those Out of Favor) for an employee who is In Good Standing at the issue of the policy.
iii. Calculate the total reserve that Italian Life should hold at the end of the second year for a policy that was issued to an employee who was In Good Standing.
iv. Calculate the actuarial present value of the death benefit for an employee who is Out of Favor at the issue of the policy.
v. For an employee who is Out of Favor when the policy is issued, the annual contract premium payable at the beginning of each year that the employee is not Dead is 3500. Calculate the actuarial present value of the expected profit for Italian Life. The actuarial present value of the expected profit is the actuarial present value of the contract premiums less the actuarial present value of the death benefits.

125. A person working for Organized Crime Incorporated (OCI) can have one of three statuses during their employment. The three statuses are:

d. In Good Standing
e. Out of Favor
f. Dead

The following transitional probabilities indicates the probabilities of moving between the three status in an given year:

	In Good Standing	Out of Favor	Dead
In Good Standing	0.6	0.3	0.1
Out of Favor	0.2	0.3	0.5
Dead	0	0	1.0

The Italian Life Insurance Company issues a special four year annuity covering employees of OCI. The annuity pays a benefit of 100,000 at the end of a year if the employee is In Good Standing at the end of the year. It pays a benefit of 50,000 if an employee is Out of Favor at the end of a year. No benefit is paid if the employee is Dead at the end of a year.

Assume that the interest rate is 25% (remember who we are dealing with).

i. Calculate the actuarial present value of the annuity benefit for an employee who is In Good Standing at the issue of the policy.
ii. Calculate the annual benefit premium (paid at the beginning of the year by those in Good Standing and those Out of Favor) for an employee who is In Good Standing at the issue of the policy.
iii. Calculate the reserve that Italian Life should hold at the end of the second year for a policy that was issued to an employee who was In Good Standing.

126. You are given the following table where decrement (1) is death, decrement (2) is lapse, and decrement (3) is diagnosis of critical illness:

x	$q_x^{(1)}$	$q_x^{(2)}$	$q_x^{(3)}$	$q_x^{(\tau)}$	$p_x^{(\tau)}$	$l_x^{(\tau)}$	$d_x^{(1)}$	$d_x^{(2)}$	$d_x^{(3)}$
55	0.02	0.15	0.010						
56	0.03	0.06	0.015						
57	0.04	0.04	0.020						
58	0.05	0.03	0.025						
59	0.06	0.02	0.030						

a. Complete the table using a radix of 10,000.

b. Calculate:

i. ${}_3p_{55}^{(\tau)}$

ii. ${}_2q_{56}^{(2)}$

iii. ${}_{1|2}q_{55}^{(3)}$

iv. The probability that a person age 55 will decrement from death or critical illness before age 60.

c. Assuming uniform distribution of each decrement between integer ages, calculate:

i. ${}_{0.25}q_{55}^{(2)}$

ii. ${}_{0.5}p_{56}^{(\tau)}$

iii. ${}_{0.5}p_{56.8}^{(\tau)}$

iv. ${}_{0.5}q_{55.6}^{(1)}$

d. Assuming a constant force of decrement for each decrement between integer ages, calculate:

i. ${}_{0.25}q_{55}^{(2)}$

ii. ${}_{0.5}p_{56}^{(\tau)}$

iii. ${}_{0.5}p_{56.8}^{(\tau)}$

iv. ${}_{0.5}q_{55.6}^{(1)}$

127. A fully discrete 3 year term pays a benefit of 1000 upon any death. It pays an additional 1000 (for a total of 2000) upon death from accident. You are given:

x	$q_x^{(1)}$	$q_x^{(2)}$
20	0.030	0.010
21	0.025	0.020
22	0.020	0.030

Decrement (1) is death from accidental causes while decrement (2) is death from non-accidental causes.

The annual effective interest rate is 10%.

a. Calculate the level annual net premium for this insurance.
b. Calculate the net premium reserve at the end of year 0, 1, 2, and 3.

128. You are given:

a. $q_x'^{(1)} = 0.200$
b. $q_x'^{(2)} = 0.080$
c. $q_x'^{(3)} = 0.125$

Assuming that each decrement is uniformly distributed over each year of age in the associated single decrement table, calculate $q_x^{(1)}$.

129. You are given:

a. $q_x'^{(1)} = 0.200$
b. $q_x'^{(2)} = 0.080$
c. $q_x'^{(3)} = 0.125$

Assuming that each decrement in the multiple decrement table is uniformly distributed over each year of age, calculate $q_x^{(1)}$.

130. You are given the following for a double decrement table:

a. $q_x'^{(1)} = 0.200$
b. $q_x'^{(2)} = 0.080$

Assuming that each decrement in the multiple decrement table is uniformly distributed over each year of age, calculate ${}_{0.4}q_{x+0.4}^{(1)}$.

131. You are given:

a. $q_x'^{(1)} = 0.200$

b. $q_x'^{(2)} = 0.080$

Decrement (1) is uniformly distributed over the year. Decrement (2) occurs at time 0.6.

Calculate $q_x^{(2)}$.

132. For a double decrement table with $l_{40}^{\tau} = 2000$:

x	$q_x^{(1)}$	$q_x^{(2)}$	$q_x'^{(1)}$	$q_x'^{(2)}$
40	0.24	0.10	0.25	y
41	--	--	0.20	2y

Calculate l_{42}^{τ}.

133. For iPhones, the phone may cease service for mechanical failure or for other reasons (lost, stolen, dropped in a pitcher of beer, etc). You are given the following double decrement table:

	For an iPhone at the beginning of the year of service, probability of		
Year of Service	Mechanical Failure	Failure for Other Reasons	Survival through the year of service
1	0.2	0.30	--
2	--	0.40	--
3	--	--	0.20

You are also given:

a. The number of iPhones that terminate for other reasons in year 3 is 40% of the number of iPhones that survive to the end of year 2.

b. The number of iPhones that terminate for other reasons in year 2 is 80% of the number of iPhones that survive to the end of year 2.

Calculate the probability that an iPhone will cease to function due to mechanical failure during the three year period following its entry into service.

134. *Your actuarial student has constructed a multiple decrement table using independent mortality and lapse tables. The multiple decrement table values, where decrement d is death and decrement w is lapse, are as follows:

$l_{60}^{(\tau)}$	$d_{60}^{(d)}$	$d_{60}^{(w)}$	$l_{61}^{(\tau)}$
950,000	2,580	94,742	852,678

You discover that an incorrect value of $q_{60}'^{(w)}$ was taken from the independent lapse table. The correct value is 0.05.

Decrements are uniformly distributed over each year of age in the multiple decrement table.

You correct the multiple decrement table, keeping $l_{60}^{(\tau)} = 950,000$.

Calculate the correct values of $d_{60}^{(w)}$.

135. You are given that mortality follows the Illustrative Life Table with $i = 0.06$. Assuming that lives are independent and that deaths are uniformly distributed between integral ages, calculate:

a. ${}_{10}q_{50:60}$

b. ${}_{10|}q_{50:60}$

c. $A_{60:60}$

d. The net annual premium for a fully discrete joint whole life on (60) and (60) with a death benefit of 1000.

e. $\bar{A}_{60:60}$

f. The net annual premium rate for a fully continuous joint whole life on (60) and (60) with a death benefit of 1000.

g. ${}_{10}p_{\overline{50:60}}$

h. Calculate the probability that the survivor of (50) and (60) dies in year 11.

i. Calculate the probability that exactly one life of (50) and (60) is alive after 10 years.

j. $A_{\overline{60:70}}$

k. The net annual premium rate for a fully discrete survivor whole life on (60) and (70) with a death benefit of 1000.

l. $\ddot{a}_{\overline{60:60}}$

m. $\ddot{a}_{\overline{60:70}}$

n. $\ddot{a}_{70|60}$

136. An joint annuity on (50) and (60) pays a benefit of 1 at the beginning of each year if both annuitants are alive. The annuity pays a benefit of 2/3 at the beginning of each year if one annuitant is alive.

You are given:

i. Mortality follows the Illustrative Life Table.
ii. (50) and (60) are independent lives.
iii. $i = 0.06$

Calculate the actuarial present value of this annuity.

137. An joint annuity on (50) and (60) pays a benefit of 1 at the beginning of each year if both annuitants are alive. The annuity pays a benefit of 2/3 at the beginning of each year if only (50) is alive. The annuity pays a benefit of ½ at the beginning of each year if only (60) is alive.

You are given:

i. Mortality follows the Illustrative Life Table.
ii. (50) and (60) are independent lives.
iii. $i = 0.06$

Calculate the actuarial present value of this annuity.

138. You are given the following mortality table:

x	l_x	q_x	p_x
90	1000	0.10	0.90
91	900	0.20	0.80
92	720	0.40	0.60
93	432	0.50	0.50
94	216	1.00	0.00
95	0		

Assume that deaths are uniformly distributed between integral ages and the lives are independent. Calculate at $i = 4\%$:

a. $A_{\overline{91:92}}$

b. $A_{\overset{1}{\overline{90:91}}:\overline{3}|}$

c. $\ddot{a}_{92:93}$

139. * You are given:

i. ${}_{3}p_{40} = 0.990$
ii. ${}_{6}p_{40} = 0.980$
iii. ${}_{9}p_{40} = 0.965$
iv. ${}_{12}p_{40} = 0.945$
v. ${}_{15}p_{40} = 0.920$
vi. ${}_{18}p_{40} = 0.890$

For two independent lives aged 40, calculate the probability that the first death occurs after 6 years, but before 12 years.

140. * For a last survivor insurance of 10,000 on independent lives (70) and (80), you are given:

i. The benefit, payable at the end of the year of death, is paid only if the second death occurs during year 5.
ii. Mortality follows the Illustrative Life Table
iii. $i = 0.03$

Calculate the actuarial present value of this insurance.

141. * For a temporary life annuity-immediate on independent lives (30) and (40):

i. Mortality follows the Illustrative Life Table
ii. $i = 0.06$

Calculate $a_{30:40:\overline{10}|}$

142. Two lives, (x) and (y), are subject to a common shock model. You are given:

i. $\mu_x(t) = 0.04$

ii. $\mu_y(t) = 0.06$

iii. $\mu_x(t)$ and $\mu_y(t)$ do not reflect the mortality from the common shock.

iv. The mortality from the common shock is a constant force of 0.01.

v. $\delta = 0.03$

Calculate:

a. $s_{T^*(x)}(t)$

b. $s_{T^*(y)}(t)$

c. $s_{T(x)}(t)$

d. $s_{T(y)}(t)$

e. $s_{T(x)T(y)}(t)$

f. $\mathring{e}_x$

g. $\mathring{e}_y$

h. $\mathring{e}_{xy}$

i. $\mathring{e}_{\overline{xy}}$

j. $\bar{A}_{xy}$

k. $\bar{A}_{\overline{xy}}$

l. $\bar{a}_{xy}$

m. $\bar{a}_{\overline{xy}}$

143. You are given:

i. $\mu_x(t) = 0.02t$
ii. $\mu_y(t) = (40-t)^{-1}$
iii. The lives are subject to an common shock model with $\mu = 0.015$
iv. $\mu_x(t)$ and $\mu_y(t)$ incorporate deaths from the common shock.

Calculate ${}_{3|}q_{xy}$

144. A male age 65 and a female age 60 are subject a common shock model.

You are given:

i. Male mortality follows the Illustrative Life Table.
ii. Female mortality follows the Illustrative Life Table but for a live five years younger. In other words, the mortality for a female age 60 is equal to the mortality for a person 55 in the Illustrative Life Table.
iii. The above mortality rates do not reflect the common shock risk.
iv. Both lives are subject to a common shock based on a constant force of mortality of 0.01.
v. $i = 8\%$ - NOTE that it is not 6%.

Calculate:

a. ${}_{20}E_{65}$ where (65) is a male.
b. ${}_{20}E_{60}$ where (60) is a female.
c. ${}_{20}E_{65:60}$ where (65) is a male and (60) is a female.
d. ${}_{20}E_{\overline{65:60}}$ where (65) is a male and (60) is a female.

145. * You are pricing a special 3-year annuity-due on two independent lives, both age 80. The annuity pays 30,000 if both persons are alive and 20,000 if only one person is alive.

You are given that $i = 0.05$, ${}_1p_{80} = 0.91$, ${}_2p_{80} = 0.82$ and ${}_3p_{80} = 0.72$.

Calculate the actuarial present value of this annuity.

146. *For a special continuous joint life annuity on (x) and (y), you are given:

i. The annuity payments are 25,000 per year while both are alive and 15,000 per year when only one is alive.

ii. The annuity also pays a death benefit of 30,000 upon the first death.

iii. $i = 0.06$

iv. $\bar{a}_{xy} = 8$

v. $\bar{a}_{\overline{xy}} = 10$

Calculate the actuarial present value of this special annuity.

Chapter 10

147. You are given the following mortality table:

x	q_x
100	0.20
101	0.30
102	0.45
103	0.70
104	1.00

You are given the follow term structure of interest rates:

t	y_t
1	3.50%
2	4.00%
3	4.40%
4	4.75%
5	5.00%

Using this information, calculate:

a. $1000A_{100}$
b. $\ddot{a}_{100}$
c. The annual premium that would be paid for a fully discrete whole life of 1000 on (100).
d. The one year forward rates - $f(0,1); f(1,2); f(2,3); f(3,4);$ *and* $f(4,5)$.
e. The reserve at time t=0, 1, 2, 3, 4, and 5.
f. $f(2,5)$

148. Let the random variable $D(N)$ be the number of deaths that occur in the next year if there are N lives insured. Assumes that the probability of death in the next year is 10%.

Calculate:

a. $E[D(N)]$
b. $Var[D(N)]$
c. N so that standard deviation $D(N)$ is equal to 10% of $E[D(N)]$
d. If $N = 1000$, the probability that the number of deaths will exceed 110. Assume that normal distribution holds.

149. Let the random variable $D(N)$ be the number of deaths that occur in the next year if there are N lives insured.

Assumes that the probability of death in the next year is distributed as follows:

Probability of Death	Distribution
8%	0.25
10%	0.50
12%	0.25

Calculate:

a. $E[D(N)]$
b. $Var[D(N)]$
c. If $N = 1000$, the probability that the number of deaths will exceed 110. Assume that normal distribution holds.

150. You are given the following mortality table:

x	l_x	q_x	p_x
90	1000	0.10	0.90
91	900	0.20	0.80
92	720	0.40	0.60
93	432	0.50	0.50
94	216	1.00	0.00
95	0		

a. Calculate the net annual premium for a fully discrete whole life of 100,000 on (91) at 4%.

b. Assume that the actual interest rate is distributed as follows:

Interest Rate	Probability
3.5%	0.3
4.0%	0.4
4.5%	0.3

Calculate $E[L_0]$ and $Var[L_0]$ where L_0 is the loss at issue random variable.

Chapter 11

151. You are given the following profit signature:

Time	Profit Signature	Premiums
0	-200	400
1	40	350
2	100	300
3	100	250
4	60	

a. Calculate the internal rate of return.
b. Calculate the net present value at 12%.
c. Calculate the discounted payback period at 8%.
d. Calculate the profit margin at 12%.

152. You are given the following profit vector for a policy issued to (x):

Time	Profit Vector
0	-200
1	40
2	100
3	100
4	60

The mortality rate for is $q_{x+t} = 0.03 + 0.01t$ for $t = 0,1,2,3$.

Calculate the Expected Present Value of Future Profits at 10%.

153. You are given the following for a five year term on (50):

a. The gross premium payable annually is 300.
b. The death benefit is 32,000 payable at the end of the year of death.
c. Mortality follows the Illustrative Life Table
d. The interest rate earned will be 7%.
e. The reserves in all years are zero.
f. The initial expenses at the start of the first year are 200 per policy plus 10% of premium.

g. The renewal expenses at the start of years 2 and later are 20 per policy and 5% of premium.

Calculate the profit vector for this policy.

154. You are given the following for a five year term on (65):

a. The gross premium payable annually is 300.
b. The death benefit is 10,000 payable at the end of the year of death.
c. Mortality follows the Illustrative Life Table
d. The interest rate earned will be 6%.
e. The profit at the end of the first year is 43.84.
f. The initial expenses at the start of the first year are 100 per policy plus 10% of premium.
g. The renewal expenses at the start of years 2 and later are 20 per policy and 5% of premium.

Calculate the reserve per policy at the end of the first year.

155. You are completing a profit test on a whole life issued to (60). Further , you are given:

a. The gross premium payable annually is 3000.
b. The expenses in the 10 policy year are 258.
c. The interest rate used in the profit test is 5%.
d. The reserve per policy at the end of the 9^{th} policy year is 10,000.
e. The reserve per policy at the end of the 10^{th} policy year is 11,500.
f. Mortality follows the Illustrative Life Table.
g. The profit in the 10^{th} policy year is 200 for each policy that was in force at the start of the policy year.

Calculate the amount of the death benefit.

156. *For a fully discrete whole life policy on (50) with a death benefit of 100,000, you are given:

a. Reserves equal benefit reserves calculated using the Illustrative Life Table at 6%.
b. The gross premium equals 120% of the benefit premium calculated using the Illustrative Life Table at 6%.
c. Expected expenses equal 40 plus 5% of gross premium, payable at the beginning of the year.
d. Expected mortality equals 70% of the Illustrative Life Table.
e. The expected interest rate is 7%.

Calculate the expected profit in the eleventh policy year, for a policy in force at the beginning of the year.

157.

Answers

1.

a. $\left(1-\frac{t}{125}\right)^{\frac{1}{5}}$

b. $1-\left(1-\frac{t}{125}\right)^{\frac{1}{5}}$

c. $\left(1-\frac{t}{125}\right)^{\frac{1}{5}}$

d. $\left(\frac{125-x-t}{125-x}\right)^{\frac{1}{5}}$

e. 0.95635

f. 0.87055

g. 0.07354

h. 125

i. $\frac{1}{625-5x}$

j. 0.002

k. 0.008

l. $\left(\frac{125-x-t}{125-x}\right)^{\frac{1}{5}}$

m. 0.97179

n. $1-\left(\frac{125-x-t}{125-x}\right)^{\frac{1}{5}}$

o. 0.02821

p. 1

q. 0.99732

r. $\frac{(125-x-u)^{\frac{1}{5}}-(125-x-u-t)^{\frac{1}{5}}}{(125-x)^{\frac{1}{5}}}$

s. $\frac{1}{625-5x}\left(\frac{125-x-t}{125-x}\right)^{\frac{-4}{5}}$

t. $\frac{5(125-x)}{6}$

u. $\frac{5(125-x)}{6}$

v. $\frac{25(125-x)^2}{396}$

w. 18.84446

x. 3.41656

y. 1.22830

2.

a. $(0.00027)(1.1)^x$

b. 0.0029254

c. 3.72077

d. $\exp\left(\frac{-0.00027}{\ln(1.1)}(1.1^t - 1)\right)$

e. $1 - \exp\left(\frac{-0.00027}{\ln(1.1)}(1.1^t - 1)\right)$

f. $\exp\left(\frac{-0.00027}{\ln(1.1)}(1.1^t - 1)\right)$

g. $\exp\left(\frac{-0.00027}{\ln(1.1)}(1.1)^x(1.1^t - 1)\right)$

h. 0.97252

i. 0.028088

j. 0.71135

k. ∞

l. $\exp\left(\frac{-0.00027}{\ln(1.1)}(1.1)^x(1.1^t - 1)\right)$

m. 0.58860

n. $1 - \exp\left(\frac{-0.00027}{\ln(1.1)}(1.1)^x(1.1^t - 1)\right)$

o. 0.41140

p. 1

q. 0.96729

r. $\exp\left(\frac{-0.00027}{\ln(1.1)}(1.1)^x(1.1^u - 1)\right) - \exp\left(\frac{-0.00027}{\ln(1.1)}(1.1)^x(1.1^{u+t} - 1)\right)$

s. $\exp\left(\frac{-0.00027}{\ln(1.1)}(1.1)^x(1.1^t - 1)\right) \bullet (0.00027)(1.1)^{x+t}$

3.

a. $\exp(-tc)$

b. $1 - \exp(-tc)$

c. ∞

d. $\frac{1}{c}$

e. $\frac{1}{c^2}$

f. $\frac{1}{e^c - 1}$

g. e^{-10c}

h. e^{-10c}

i. e^{-10c}

4.

a. $\dfrac{x^2}{10,000}$

b. $1-\dfrac{x^2}{10,000}$

c. $\dfrac{10,000-(x+t)^2}{10,000-x^2}$

d. $\dfrac{x}{5000}$

e. 66.66667

f. 555.56

g. 0.16

h. 0.84

i. 0.20

j. $\dfrac{2x}{10,000-x^2}$

k. $\dfrac{6}{175}$

l. $\dfrac{10,000-(x+t)^2}{10,000-x^2}$

m. $\dfrac{2xt+t^2}{10,000-x^2}$

n. $\dfrac{150t+t^2}{4375}$

o. $\dfrac{4375-150t-t^2}{4375}$

p. $\dfrac{(100-x)(200+x)}{300+3x}$

q. 13.09524

r. $\dfrac{(100-x)(200+x)}{300+3x}$

s. 66.66667

t. 13.09524

u. 8.20952

v. $\dfrac{2(x+u)t+t^2}{10,000-x^2}$

w. 0.2

5. $F_0(x)=1-\left(\dfrac{100-x}{100}\right)^2$ *and* ${}_{10}p_{50}=0.64$

6.

a. 0.98

b. 0.97515

c. 0.96939

d. 0.95969

e. 0.03031

7.

a. No answer provided

b. 6.432

c. 125.89 for non-smokers and 80.11 for smokers

8.

a. 0.3675

b. 1.00168

c. 2.04000

9. 37.11434

10.

a. 0.93132
b. 0.06868
c. 0.42737
d. 0.43703
e. 0.56297
f. 0.24087
g. 0.91970

11.

a. 0.04015
b. 0.95985
c. 0.08366
d. 0.87940
e. 0.12060
f. 0.04183
g. 0.04097
h. 0.19042

12.

a. 0.04099
b. 0.95901
c. 0.08371
d. 0.87847
e. 0.12153
f. 0.04099
g. 0.04099
h. 0.19013

13.

a. 0.03
b. 0.030464
c. 0.03785
d. 0.03829

14.

x	q_x	l_x	d_x
50	0.040	20,000	800
51	0.0625	19,200	1,200
52	0.075	18,000	1,350
53	0.100	16,650	1,665
54	0.125	14,985	1,873.125

15. $1/3$

16.

a. 0.960
b. 0.951
c. 0.943
d. 0.935
e. 0.935
f. 0.63944
g. 0.11594
h. 0.60680
i. 0.04393

17. 0.20
18. 0.0782
19. 1.7202 and 1.3993
20. 0.10754
21. 103,973.04
22. 1800.355
23.

$[x]$	$l_{[x]}$	$l_{[x]+1}$	l_{x+2}
105	1650	1600	1500
106	1518	1411.765	1200
107	1185.185	1066.667	800
108	768	640	400
109	304.762	228.571	100

24. 99,798.4
25. 12.11066

25.5 Age 30 E[Z]= 303.89; Var[Z]= 412,400; Age 50 E[Z]= 803.53; Var[Z]= 1,014,000

26.

a. 0.90668
b. 0.70152
c. 0.21336
d. 914.88
e. 0.41851
f. 0.00142
g. 0.92017
h. 960.21411
i. 2.24471
j. 1.42410

27.
   a. 0.24905
   b. 0.13037
   c. 0.36084
   d. 0.11868
   e. 0.22388
   f. 0.03273
   g. 0.05574
   h. 0.44195
28. $i = 11.11111\%$ and $q_x = 0.10$
29. 0.70000
30. 52,965.3
31.
   a. 256.45
   b. 134.24
   c. 364.71
   d. 122.21
   e. 254.59
32.
   a. 256.41
   b. 134.22
   c. 364.69
   d. 122.19
   e. 254.55
33.
   a. 0.14262
   b. 0.84643
34. 0.80691
35. 1
36. 912.59
37. 667.78500 and 858,389.55
38. 0.66575
39. 1730.10
40. 6688.26
41. 12.27
42. 15,513.77
43. 10,417
44. 466,262
45. 77.94%
46. 62.45%

47.
    c. 5.07307
48. 0.59704
49. 0.01
50.
    a. 2.42638
    b. 1.42638
    c. 2.21302
    d. 0.96334
    e. 3.09945
    f. 0.53197
    g. 0.65715
    h. 1.9200
    i. 1.03247
    j. 4.72326
    k. 3.86982
51.
    a. 11.1454
    b. 10.1454
    c. 3.86647
    d. 10.26520
    e. 5.39678
    f. 10.90248
    g. 11.6682
    h. 12.0134
    i. 10.6800
    j. 10.6804
    k. 10.6811
    l. 12.8443
    m. 8.22550
52. 2019.23
53. 103,990.63
54.
    a. 2.71307
    b. 2.41688
55. 3.77541
56. 4.0014%
57. 0.98220
58.
    a. 3.66643
    b. 3.45057
    c. 8.37502

d. 3.16305
e. 119.14
f. 0.00421

59. 612.415
60. 15
61.
    a. 75,587.92
    b. 75,628.42
62. 114.42
63. 0.1293
64. 0.9285
65.
    a. 2221.91
    b. $L_0^n = 50,000v^{K_x+1} - 2221.91\ddot{a}_{\overline{K_x+1}|}$
    c. 339,408,907
    d. 194.29
66.
    a. 4539.02
    b. $L_0^n = 50,000v^{T_x} - 4539.02\bar{a}_{\overline{T_x}|}$
    c. 642,744,265
67.
    a. 515.22
    b. $L_0^n = 25,000v^{\min(K_x+1,25)} - 515.22\ddot{a}_{\overline{\min(K_x+1,25)}|}$
    c. 12,723,898
    d. 44.53
68. 293.84
69. 189,127
70. 7252.05
71.
    a. 1420.73
    b. 3,360,293
    c. 218.85
72.
    a. 216,326.38
    b. 13,731.03
    c. 0.0052
73. 4639.89
74. 382.84
75. 488.03
76. 1601.68

77. 12.5%
78. -881.44
79. 64,848.48
80. 455.66
81.
    a. 2160.10
    b. 0.70230
    c. 2201.76
82.
    a. 3859.18333
    b. See Solutions
    c. 17,024,079
    d. -341.68 and 18,183,745
83. 16,231.09
84.
    a. 23,113.91
    b. 5444.04
    c. 1481.29 and 3448.02
    d. 7217 and 6217
    e. 11,900.31
    f. 12,092.84
    g. 21,412.35
    h. 5005.50
    i. 1337.23 and 3448.02

85.

a. 21,867.19
b. – 1199.75 and 4179.77
c. 2166.04 and 6003.56

86. 166.67
87. 300
88. 2357.78, 3897.20, 5878.63, 0
89. 225.38
90. 0, 1082.33, 1640.84, 0
91. 7/55
92. 90
93. 2407.37
94. 303.05
95.

a. 1714.61397
b. 3052.20
c. 3866.53
d. 36.47
e. 57.44, -9.84, and -11.13

96. -6213
97. 0, 18.99, 138.26, 219.39
98. 95.96
99. 324.70
100. 77.66
101.

i. 44,438.16
ii. 48,437.44
iii. 3999.28
iv. 270,779.80
v. -17,246.96
vi. 253,532.84
vii. 3250.89
viii. 30,517.00

102.

i. 231.01
ii.

t	${}_tV^b$	${}_tV^e$	${}_tAS$
0	0	0	0
1	47.50	-88.71	-41.22
2	49.05	-46.39	2.65
3	0	0	0

103. 2415
104. 16,932.08
105. 5926.76
106. 13,460.86
107.
   a. 1298.1132
   b. 3510.5144
   c. 21,412.35
   d. 389.43
   e. 24,201.92
108.
   a. 1311.3208
   b. 2671.9534
   c. 10,011.00
109.
   a. 402.2642
   b. 1056.2224
   c. 6715.85
110. 350.07
111. 43,386.05
112.
   a. 0.27041
   b. 0.72959
   c. 0.00000
113.
   a. 0.54881
   b. 0.33740
   c. 0.11379
114.
   a. 0.54881
   b.
      i. 1.00000
      ii. 0.00000
      iii. 0.99500
      iv. 0.004167
      v. 0.990035
      vi. 0.008295
115. 0.074238
116. 0.0209
117. 0.1202
118. 0.1539
119.

i. 621.1
ii. 235.33479

120. 0.35125

121. 15

122. 55

123.

i. 71.140.12
ii. 37,838.09
iii. 46,760.15
iv. 12,038.07
v. -5,560.92
vi. 105,899.42

124.

i. 3599.51
ii. 1484.79
iii. 928.08
iv. 5823.44
v. 240.46

125.

i. 128,854.27
ii. 53,152.09
iii. -9,510.99

126.

a.

x	$q_x^{(1)}$	$q_x^{(2)}$	$q_x^{(3)}$	$q_x^{(\tau)}$	$p_x^{(\tau)}$	$l_x^{(\tau)}$	$d_x^{(1)}$	$d_x^{(2)}$	$d_x^{(3)}$
55	0.02	0.15	0.010	0.180	0.820	10,000	200	1500	100
56	0.03	0.06	0.015	0.105	0.895	8,200	246	492	123
57	0.04	0.04	0.020	0.100	0.900	7,339	293.56	293.56	146.78
58	0.05	0.03	0.025	0.105	0.895	6,605.1	330.255	198.153	165.1275
59	0.06	0.02	0.030	0.110	0.890	5,911.5645	354.69387	118.23129	177.346935

b.

i. 0.66051
ii. 0.0958
iii. 0.026978
iv. 0.21368

c.

i. 0.0375
ii. 0.9475
iii. 0.94776

iv. 0.01173

d.

i. 0.04034
ii. 0.94604
iii. 0.94763
iv. 0.01139

127.

a. 63.64
b. The reserve is zero at the end of each year.

128. 0.180167

129. 0.180520

130. 0.085951

131. 0.0704

132. 802.56

133. 0.35

134. 47,433

135.

a. 0.26084
b. 0.03436
c. 0.47975
d. 52.20
e. 0.49400
f. 56.89
g. 0.98364
h. 0.00504
i. 0.24448
j. 0.31180
k. 25.65
l. 13.0997
m. 12.1584
n. 3.5891

136. 12.8768

137. 12.7182

138.

a. 0.93867
b. 0.83045
c. 1.28846

139. 0.067375

140. 234.82

141. 7.1687

142.

a. $e^{-0.05t}$
b. $e^{-0.07t}$
c. $e^{-0.11t}$
d. 20
e. 14.286
f. 9.091
g. 25.195
h. 0.78571
i. 0.53929

143. 0.06994

144.
a. 0.05498
b. 0.10970
c. 0.03434
d. 0.13034

145. 80,431.70

146. 246,015.46

147.
a. 888.06
b. 2.6381
c. 336.63
d. 3.5%; 4.5024%; 5.2046%; 5.8071%; and 6.0060%
e. 0; 185.51; 350.93; 496.98; 606.72; and 0
f. 5.672%

148.
a. 0.1N
b. 0.09N
c. 900
d. 14.69%

149.
a. 0.1N
b. $0.0898N + 0.0002N^2$
c. 27.76%

150.
a. 37,367.56
b. 4.37 and 1,636,411,365

151.
a. 17.4%
b. 24.74
c. 3 Years
d. 2.19%

152. 22.74

153. -230, 115.46, 77.65, 59.77, 40.20, 19.34

154. 54.32

155. 70,983.70

156. 756